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Some remarks on the reconstruction problems of symplectic and cosymplectic manifolds

Agnieszka KOWALIK

1. Introduction

The presentation will contain some results concerning reconstruction problems of symplectic and cosymplectic manifolds.
In both cases following theorems of M. Rubin will be used:

Theorem 1.1 (M. Rubin [2]). *Let X and Y be regular topological spaces and let $\mathrm{H}(X)$, $\mathrm{H}(Y)$ denote groups of all homeomorphisms on X, Y respectively. Let $G \leq \mathrm{H}(X)$ and $H \leq \mathrm{H}(Y)$ be factorizable and non-fixing. Assume that there is an isomorphism $\varphi : G \to H$. Then there is a unique homeomorphism $\tau : X \to Y$ such that for any $g \in G$ one have $\varphi(g) = \tau g \tau^{-1}$.*

Theorem 1.2 (M. Rubin [2]). *Let X, Y be regular topological spaces and let $G \leq \mathrm{H}(X)$, $H \leq \mathrm{H}(X)$. Assume that*

1. *There are $G_1 \leq G$ and $H_1 \leq H$ such that G_1, H_1 are factorizable and non-fixing groups of X and Y respectively.*
2. *For every $x \in X$, $\mathrm{int}\overline{G(x)} \neq \emptyset$ and for every $y \in Y$, $\mathrm{int}\overline{H(y)} \neq \emptyset$.*

Suppose that there is a group isomorphism $\varphi : G \to H$. Then there is a homeomorphism $\tau : X \to Y$ such that $\varphi(g) = \tau g \tau^{-1}$ for any $g \in G$.

In the case of symplectic manifold (M, ω) the symbol $\mathrm{Symp}(M, \omega)$ will stand for the group of all symplectomorphisms on (M, ω). In the case of cosymplectic manifold (M, θ, ω) symbols $\mathrm{Cosymp}(M, \theta, \omega)$, $\mathrm{Ham}(M, \theta, \omega)$, $\mathrm{Grad}(M, \theta, \omega)$ and $\mathrm{Ev}(M, \theta, \omega)$ will stand for the groups of all cosymplectomorphisms and hamiltonian, gradient and evolution cosymplectomorphisms respectively. In both symplectic and cosymplectic cases if G is a group then G_c denotes its subgroup of all compactly supported elements and G_0 denotes its subgroup of all elements that are isotopic with the identity.

2. Main results

Our first result is an extension of theorem of A. Banyaga [1]. This can be done by using Thorem 1.2. Using it we obtain the following:

Theorem 2.1. *Let* (M_i, ω_i) *for* $i = 1, 2$ *be symplectic manifolds and let* $\varphi : \mathrm{Symp}(M_1, \omega_1) \to \mathrm{Symp}(M_2, \omega_2)$ *or* $\varphi : \mathrm{Symp}(M_1, \omega_1)_0 \to \mathrm{Symp}(M_2, \omega_2)_0$ *be an isomorphism. Then there is a unique diffeomorphism* $\tau : M_1 \to M_2$ *such that* $\varphi(f) = \tau f \tau^{-1}$ *for any* $f \in \mathrm{Symp}(M_1, \omega_1)$ *and* $\tau^*\omega_2 = \lambda\omega_1$ *for some constant* λ.

In [1] one can find a similar result, but with stronger assumptions. Namely it must be fullfiled that both M_i are compact or that stmplectic pairing for both ω_i is identically equal to zero.

Our next results deal with cosymplectic manifolds. Among the others we obtain an analogon of above theorem for cosymplectic manifolds.
Our first result is the following:

Proposition 2.2. *Groups* $\mathrm{Ham}(M, \theta, \omega)$ *and* $\mathrm{Grad}(M, \theta, \omega)$ *are factorizable and non-fixing.*

By using above Proposition and Theorem 1.1 we obtain immediately:

Corollary 2.3. *Let* $(M_1, \theta_1, \omega_1)$ *and* $(M_2, \theta_2, \omega_2)$ *be cosymplectic manifolds and let* $G(M_i) = \mathrm{Ham}_c(M_i, \theta_i, \omega_i)$ *or* $G(M_i) = \mathrm{Grad}_c(M_i, \theta_i, \omega_i)$ *for* $i = 1, 2$. *If there is an isomorphism* $\varphi : G(M_1) \to G(M_2)$ *then there is a unique homeomorphism* $\tau : M_1 \to M_2$ *such that for any* $g \in G(M_1)$ *one have* $\varphi(g) = \tau g \tau^{-1}$.

Our next result is the following extension of Theorem of Takens [4] to the cosymplectic case.

Theorem 2.4. *Let* $(M_1, \theta_1, \omega_1)$ *and* $(M_2, \theta_2, \omega_2)$ *be cosymplectic manifolds with complete Reeb vector fields. Let*

$$\tau : (M_1, \theta_1, \omega_1) \to (M_2, \theta_2, \omega_2)$$

be a homeomorphism such that

$$\tau h \tau^{-1} \in \mathrm{Cosymp}(M_2) \Leftrightarrow h \in \mathrm{Cosymp}(M_1)$$

or

$$\tau h \tau^{-1} \in \mathrm{Grad}(M_2, \theta_2, \omega_2) \Leftrightarrow h \in \mathrm{Grad}(M_1, \theta_1, \omega_1),$$

or

$$\tau h \tau^{-1} \in \mathrm{Ev}(M_2, \theta_2, \omega_2) \Leftrightarrow h \in \mathrm{Ev}(M_1, \theta_1, \omega_1).$$

Then τ is a $\mathcal{C}^\infty$ diffeomorphism.

Theorem 2.5. *Let $(M_1, \theta_1, \omega_1)$ and $(M_2, \theta_2, \omega_2)$ be cosymplectic manifolds. Let $G(M_i)$ be either $G(M_i) = \mathrm{Cosymp}(M_i, \theta_i, \omega_i)$ or $G(M_i) = \mathrm{Ev}(M_i, \theta_i, \omega_i)$ or $G(M_i) = \mathrm{Grad}(M_i, \theta_i, \omega_i)$ for $i = 1, 2$. If there exists an isomorphism $\varphi : G(M_1) \to G(M_2)$ then there is a unique smooth diffeomorphism $\tau : M_1 \to M_2$ such that for any $f \in G(M_1)$ there is $\varphi(f) = \tau f \tau^{-1}$ and $\tau^* \omega_1 = \lambda \omega_2$.*

References

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AGH University of Science and Technology, Cracow, Poland
al. Mickiewicz 30 30-059 Cracow, Poland
E-mail: kowalik@wms.mat.agh.edu.pl